Difference-in-Differences Analysis

Conference on Empirical Legal Studies

NYU Law School, November 10, 2007

Daniel Rubinfeld

Professor of Law, Professor of Economics, UC Berkeley

Visiting Professor of Law, NYU

Introduction To Difference in Differences (DD)

Data	Randomized experiment or natural experiment (quasi-experiment), both of which generate observational data
Base Case	Two or more groups, two or more periods of time. In some periods, some groups are exposed to the treatment.
Problem	We cannot observe the counterfactual (what if the treatment group had not received treatment)
Issues	Selection Bias, Omitted Variables Bias, Simultaneity

Overview of DD

Assume that treatments are randomly assigned to some units

(or a natural experiment assigns treatment “as if” by random assignment)

To estimate the treatment effect, one might compare the treated units before and after treatment

However, this might pick up the effects of other factors that changed around the time of treatment

Therefore, we use a control group to “difference out” these confounding factors and isolate the treatment effect

Differences: The Base Case

One could take the mean of each group's outcome with and without treatment (e.g., T_B = mean pre-treatment outcome)

	Treatment group
Before	T_B
After	T_A

and then calculate the "difference" of the means:

Treatment effect = $(T_A - T_B)$

Why Not Use a Differences Model in a Regression Framework

$y_i = \alpha_0 + \alpha_1 \, TRT_i + \varepsilon_i$

TRT = 1 if in treatment group, = 0 if in control group

α_1, the differences estimator, measures the treatment effect (if $E(\varepsilon_i \mid TRT) = 0$). But,

(1) What if some of the unobserved variables are persistent over time?

(2) What if there are pre-treatment differences in y among various groups?

Difference in Differences (DD) can do better.

DD: The Base Case

One could simply take the mean value of each group's outcome before and after treatment

	Treatment group	Control group
Before	T_B	C_B
After	T_A	C_A

and then calculate the "difference-in-differences" of the means:

DD = Treatment effect = $(T_A - T_B) - (C_A - C_B)$

DD in the Regression Context

Putting the same analysis in a regression framework (we can add additional covariates as appropriate):

$$y_i = \beta_0 + \beta_1 TRT_i + \beta_2 AFT_i + \beta_3 TRT_i{*}AFT_i + \varepsilon_i$$

TRT = 1 if in treatment group, = 0 if in control group

AFT = 1 if after treatment, = 0 if before

$E(\varepsilon_i \mid TRT, AFT) = 0$

The coefficient on the interaction term (β_3) measures the DD estimate of the treatment effect

The Details

To see this, look at all possible combinations (treatment-control and before-after), substituting values into the regression equation:

$$y_i \;=\; \beta_0 \;+\; \beta_1\,TRT_i \;+\; \beta_2\,AFT_i \;+\; \beta_3\,TRT_i{*}AFT_i \;+\; \varepsilon_i$$

	Treatment Group	Control Group	Difference
Before	$\beta_0 + \beta_1$	β_0	β_1
After	$\beta_0 + \beta_1 + \beta_2 + \beta_3$	$\beta_0 + \beta_2$	$\beta_1 + \beta_3$
Difference	$\beta_2 + \beta_3$	β_2	β_3

An Equivalent DD Regression Model

After: $y_i = \beta_0 + \beta_1 \, TRT_i + \beta_2 + \beta_3 \, TRT_i + \varepsilon_i$

Before: $y_i = \beta_0 + \beta_1 \, TRT_i + \varepsilon_i$

After – Before:

$$\Delta y_i = \beta_2 + \beta_3 \, TRT_i + u_i$$

Why Use DD?

Suppose that TRT is correlated with y, i.e., there are pre-treatment differences in y between those that will receive the treatment and those that will not.

If, for example, $T_B > C_B$, and $C_A > C_B$, then the Differences Estimator will mistakenly overvalue the success of the treatment, since the treatment group had better outcomes than the control group before the treatment was applied.

Example 1: Effects of the Minimum Wage: Card and Krueger (1994)

Card and Krueger (1994)

What is the effect of increasing the minimum wage on employment (E) at fast food restaurants?

Confounding factor: national recession

Treatment group = NJ
Control group = PA

Before = Feb 92
After = Nov 92

Card and Krueger (continued)

Regression Model:

$$E_i = \beta_0 \;+\; \beta_1 NJ_i \;+\; \beta_2 Nov92_i \;+\; \beta_3 NJ_i{*}Nov92_i \;+ \varepsilon_i$$

$E_i =$ 23.33 - 2.89 NJ_i - 2.16 $Nov92_i$ + 2.75 $NJ_i{*}Nov92_i$

1.35) (1.44) (1.25) (1.34)

Standard Errors are in Parentheses

Example 2: The Effect of Information Disclosure Laws on Restaurant Sales

Jin and Leslie (2003) show that posting of hygiene score cards in LA increase restaurant profitability and sales

Groups: LA restaurants and restaurants in other neighboring counties

Treatment: Posting of hygiene cards

Example 3: The Effect of Airline Alliances on Airline Fares: Bamberger, Carlton, and Neumann (2004)

- Treatment: Two Alliances: Continental/America West and Northwest/Alaska
- Controls: (i) Presence of Southwest Airlines; (ii) Percentage of Passengers Flying Direct; and (iii) Market Concentration.
- Conclusion: Average Fares fell from 5-7 percent as a result of the alliances

DD: Issues

If the treatment is not randomized, ε will not be independent of TRT and AFT. Outcomes will be affected both by the treatment and by the effect of non-random assignment.

If the effect of the treatment was not the same for all members of the treatment group (heterogeneity) due, for example, to partial compliance, the key parameter estimates will be biased (the causal effect will vary from individual to individual).

If the treatment is randomly assigned, then, DD will measure the average treatment effect. However, if the treatment is only partially randomly determined, as in many quasi-experiments, then instrumental variables estimation may be necessary.

DD: Further Issues

DD typically uses several years of serially-correlated data, but often fails to account for the fact that (absent a correction) this will overstate the statistical significance of the results (Bertrand, Duflo, and Mullainathan 2004)

What would have happened to the non-treatment group had there been no treatment? Consider adding additional covariates. Check to see if TRT is correlated with characteristics of those receiving treatment.

Suppose there was partial compliance; this can cause bias, which can be remedied through the use of Instrumental Variables estimation.

Functional form may matter (e.g., consider testing between logarithmic and linear functions).

Difference-in-Difference-in-Differences (DDD)

If one is concerned that the estimated treatment effect might be spurious, a useful test involves finding a comparison group that should not be affected by the treatment.

Suppose a state implements a health policy law that is aimed at improving the health of the elderly (age 65 and older).

A DD model might compare outcomes before and after the policy change; the control group is the people under 65 and the treatment group those 65 or older.

But, suppose that federal health care policy affects the elderly differently than those under 65.

An Alternative DD model might use the elderly from another state as a control group.

DDD Analysis (continued)

Rather than do separate DD analyses, we can combine the two:

Let TRT = 1 if elderly; 0 otherwise

AFT = 1 if after the law is passed; 0 if before

IN = 1 if the individual is in the treatment state; 0 if not

The DDD regression model is:

$$y_i = \beta_0 + \beta_1 TRT_i + \beta_2 AFT_i + \beta_3 TRT_i*AFT_i + \beta_4 IN_i + \beta_5 IN_i*TRT_i + \beta_6 IN_i*AFT_i + \beta_7 TRT_i*AFT_i*IN_i + \varepsilon_i$$

β_7 measures the mean differences in elderly outcomes in the treatment state after netting out the change in means for elderly out of state and the change in means for the non-elderly in the treatment state.

References

Stock, James and Mark Watson, *Introduction to Econometrics*

Bamberger, Gustavo, Dennis Carlton, and Lynette Neumann, "An Empirical Investigation of the Effects of Airline Alliances," *Journal of Law and Economics*, April, 2004.

Bertrand, Marianne, Esther Duflo, and Sendhil Mullainathan, "How Much Should We Trust Differences-in-Differences Estimates?" *Quarterly Journal of Economics,* February 2004

Card, David and Alan Krueger, "Minimum Wages and Employment: A Case Study of the Fast Food Industry in New Jersey and Pennsylvania," *American Economic Review*, September 1994

Imbens, Guido and Jeffrey Wooldridge, "What's New in Econometrics," NBER Summer Institute, 2007.

Jin, Ginger and Phillip Leslie, "The Effect of Information Disclosure on Product Quality: Evidence from Restaurant Hygiene Cards," *Quarterly Journal of Economics*, May, 2003.